We construct high-quality graphene-based van der Waals devices with narrow superconducting niobium nitride (NbN) electrodes, in which superconductivity and a robust fractional quantum Hall (FQH) state coexist. We find a possible signature for crossed Andreev reflection (CAR) across the superconductor separating two FQH edges. Our observed CAR probabilities in the particlelike fractional fillings are markedly higher than those in the integer and hole-conjugate fractional fillings and depend strongly on temperature and magnetic field unlike the other fillings. Further, we find a filling-independent CAR probability in integer fillings, which we attribute to spin-orbit coupling in NbN allowing for Andreev reflection between spin-polarized edges. These results provide a route to realize novel topological superconducting phases in FQH-superconductor hybrid devices based on graphene and NbN.

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I. INTRODUCTION

Topological superconductors are predicted to represent a phase of matter with nonlocal properties, providing a robustness suitable for quantum computing [1–5]. A theoretical proposal to synthesize a topological superconductor from a topological insulator and a conventional (s-wave) superconductor has motivated hybrid approaches to realize Majorana modes. Besides topological insulators [6–8], these approaches now include spin-orbit coupled semiconductors [9–14], magnetic atom chains [15], and integer quantum Hall edges [16–19]—all in combination with a superconductor—offering either a test bed for or a route toward topological qubits. Common to all of these is the noninteracting description of charge carriers and Ising topological order which is insufficient for universal quantum computation [4]. These approaches, however, can be extended to the computationally universal Fibonacci order [20] predicted to emerge in a coupled parafermion array [21].

Parafermions, unlike Majoranas, require electron-electron interactions to form, which result in richer non-Abelian braiding statistics [22]. An established condensed matter system that forms with interactions is the fractional quantum Hall (FQH) state, which is the basis of different approaches for synthesizing parafermions [20–29]. The primary approach—combining a FQH state, appearing in semiconductor heterostructures, with superconductivity [21,24–28]—has so far presented two major experimental challenges. First, the strong magnetic fields required for a FQH state suppress superconductivity [16–19,30]. Second, coupling a superconductor to a semiconductor heterostructure [31] can be difficult, often leading to a nontransparent interface. Here, we overcome these challenges by using graphene-based van der Waals (vdW) heterostructures coupled to superconducting niobium nitride (NbN). The high device quality decreases the magnetic fields required for a robust FQH state to the regime where NbN remains superconducting owing to its large critical field. The superconductor edge contact to graphene provides an interface transparent enough to allow Andreev reflection in quantum Hall edges.
II. DEVICE AND CROSSED ANDREEV REFLECTION IN THE FQH STATE

Figure 1(a) shows the schematic of our vdW heterostructure, consisting of single-layer graphene as the conducting channel, which is first encapsulated by hexagonal boron nitride dielectric and then by graphite on both top and bottom. This heterostructure maximizes the channel mobility owing to the metallic graphite layers screening remote impurities [32,33], which is essential for reaching the FQH phase at magnetic fields low enough to allow superconductivity. Figure 1(b) shows a typical device, including the heterostructure (purple) and a <100-nm-wide NbN superconductor (blue) which is sufficiently narrow (coherence length ~50 nm) [17] to allow the possibility of crossed Andreev reflection (CAR) between the quantum Hall edges on both sides (see also Fig. 6 for the images of the measured devices), a necessary ingredient for realizing parafermions [34]. CAR dramatically affects transport: when the injected electronlike charges are drained from the superconductor, holelike charges propagate away [Fig. 1(c)].

We have measured the resistance $R_{\text{CAR}} = V_{\text{CAR}}/I_{\text{exc}}$ as well as the Hall resistance $R_{XY} = V_{XY}/I_{\text{exc}}$ as a function of gate voltage (charge carrier density) at a magnetic

![Diagram](image1.png)

**FIG. 1.** The device and crossed Andreev reflection (CAR) in a FQH state. (a) Schematic of the heterostructure. Graphene is encapsulated with boron nitride dielectric and graphite. (b) Typical device including a NbN superconductor <100 nm in width and ~1 μm in length (blue). The extended arms connect to normal leads (not shown) that are used to bias a current $I_{\text{exc}}$, measure the voltages $V_{XX}$ and $V_{XY}$, the potential of the edge mode propagating toward the superconductor $V_\nu$, and finally that of the edge mode propagating away $V_{\text{CAR}}$. The superconductor is grounded, remaining leads are floating. The solid and the dashed arrows depict, respectively, the chiral electron and hole conduction in an out-of-plane magnetic field $B$. The metal electrodes on top graphite (gate) are bridges that connect the top gate to leads avoiding the edge of the heterostructure. (c) Illustration of the theory model. A narrow superconductor induces a pairing gap $\Delta$ between the counterpropagating fractional quantum Hall edges along both sides. $\Delta$ converts an incoming electron to an outgoing hole by crossed Andreev reflection. (d) $R_{\text{CAR}} = V_{\text{CAR}}/I_{\text{exc}}$ and $R_{XY} = V_{XY}/I_{\text{exc}}$ as a function of gate voltage measured at $B = 14$ T for different temperatures $T$. An $R_{\text{CAR}} < 0$ at fractional quantum Hall plateaus indicates hole conductance (CAR). Measured $R_{XY}$ for $\nu < 1$ (high channel impedance) is slightly lower than the expected quantized values, with up to ~10% deviation for $\nu = 1/3$, due to a resulting small low-pass frequency in this experimental setup. Similar signal magnitude reduction is expected for $R_{\text{CAR}}$ for $\nu < 1$. This effect does not result in a spurious negative contribution that can increase the negative CAR reading. Inset of (d) shows the resistance of a narrow NbN for varying $T$ which superconducts below 8 K at 14 T. (e) $R_{\text{CAR}}$ at 1.75 K from (d). (f) $R_{\text{CAR}}$ as a function of filling $\nu$ and $B$ measured at 15 mK. CAR ($R_{\text{CAR}} < 0$) is observed at 3 T for $\nu = 2/3$. 

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field $B = 14$ T for different temperatures $T$ [Fig. 1(d)]. Here, $V_{\text{CAR}}$ is the potential of the edge mode propagating away from the grounded superconductor, $V_{XY}$ is the Hall voltage, and $j_{\text{exc}}$ is the bias current [see Fig. 1(b) for the circuit]. At low $T$, $R_{\text{CAR}}$ becomes negative for quantized values of $R_{XY}$. We find an $R_{\text{CAR}} < 0$ for both integer fillings 1 and 2 [17], and importantly for several fractional fillings $1/3, 2/5, 2/3, 5/3$—our main finding [enlargement shown in Fig. 1(e)]. An $R_{\text{CAR}} < 0$ indicates that the electronlike carriers drained from the superconductor produce holelike carriers with opposite charge, a direct result of crossed Andreev reflection, which reverses the sign of the edge potential. $R_{\text{CAR}}$ acquires positive values either when $R_{XY}$ is nonquantized and the bulk of the device conducts or when superconductivity is suppressed with increasing $T$—both destroying CAR as expected. We confirm that our narrow NbN superconducts at 14 T for $T < 8$ K by measuring a strip with identical dimensions as the one coupled to the quantum Hall edges [Fig. 1(d), inset]. In a separate cooldown with $T$ reaching 15 mK, we find $R_{\text{CAR}} < 0$ for several fractional fillings for a wide range of $B$ [Fig. 1(f)]. At this low temperature, CAR in the FQH state can be observed in magnetic fields as low as 3 T (filling 2/3).

### III. Spin-Orbit Coupling (SOC)

Figure 2(a) shows $R_{\text{CAR}}$ with the accompanying longitudinal resistance $R_{XX}$ as a function of filling $\nu$. For all integer fillings $R_{XX}$, which measures bulk conduction, is much smaller in amplitude than $R_{\text{CAR}}$, linking $R_{\text{CAR}}$ strictly to the potential of the edge mode leaving the superconductor. We find a negative edge potential with consistently increasing amplitude for lower fillings, which remains negative for all integer $\nu$ and measured $B$ demonstrating the robustness of CAR [Fig. 2(a), inset].

![FIG. 2. Spin-orbit coupling. (a) $R_{\text{CAR}}$ and $R_{XX}$ as a function of filling $\nu$. All integer $\nu$ show crossed Andreev reflection ($R_{\text{CAR}} < 0$). Small $R_{XY}$ indicates negligible bulk conductance. Inset of (a) shows $R_{\text{CAR}}$ measured at $B = 2, 4$ T. (b) $R_{\text{CAR}}$ probability $p_{\text{CAR}}$ of the inset of (a). All integer $\nu$, including the spin-polarized $\nu = 1$, have a comparable $p_{\text{CAR}}$, an evidence for the pairing mechanism being the same for all integer $\nu$, enabled by spin-orbit coupling. The dashed line is the average $p_{\text{CAR}}$. $H_0$ is the Hamiltonian of the edges, $H_{\text{SC}}$ the pairing, and $H_{\text{SO}}$ the spin-orbit Hamiltonian. Z represents incoming charges tunneling to the outgoing edge mode without Andreev reflection (direct tunneling). (c) Illustration of the edges separated by a superconductor without spin-orbit coupling ($\lambda_R = 0$) for $\nu = 1, 2$, and their calculated Bogoliubov–de Gennes spectrum. No pairing gap is present ($\Delta_{\text{ind}} = 0$). Momentum $k$ in units of $l_B^{-1}$ with $l_B$ the magnetic length. Solid lines are the electronlike excitations, dashed lines the holelike. Color code indicates the spin and the direction of propagation. (d) Inclusion of spin-orbit coupling tilts the spins and enables a pairing gap $\Delta_{\text{ind}}$. The only possible pairing is $p$ wave for any integer $\nu$ irrespective of whether it is spin polarized or not. The inner edge modes pair more strongly due to their proximity to the superconductor. However, for low energy and temperature, $p_{\text{CAR}}$ is expected to be independent of the size of $\Delta_{\text{ind}}$. Here, the value of $\lambda_R$ is chosen to provide a spectral superconducting gap matching the experimentally relevant $\Delta_{\text{ind}}$ (see Supplemental Material, Table S3 [35]). (e) $p_{\text{CAR}}$ at $\nu = 2$ as a function of the incoming edge mode potential $V$ (excitation) for different gate voltages spanning the entire QH plateau region. Crossed Andreev reflection is limited to below $|eV| \sim 1$ meV. (f) Same as (e) but at $\nu = 6$. CAR is observed for $|eV| < 0.2$ meV.

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The increasing amplitude of $R_{\text{CAR}}$ with decreasing $\nu$ is connected to the decreasing number of edge modes, which increases $R_{XY}$. This dependence of $R_{\text{CAR}}$ on $\nu$ can be understood by introducing $p_{\text{CAR}} = -V_{\text{CAR}}/V$, the probability of crossed Andreev reflection, where $V$ is the potential of the incoming edge mode. The Hall voltage constrains $V_{XY} = V - V_{\text{CAR}}$, leading to the proportional relation $R_{\text{CAR}} = -R_{XY}/(1 + p_{\text{CAR}}^{-1})$. This allows us to calculate $p_{\text{CAR}}$ from the measured $R_{\text{CAR}}$, and thus to directly compare the CAR rate between different fillings. Figure 2(b) shows that $p_{\text{CAR}}$ is comparable for all integer $\nu$, including the spin-polarized $\nu = 1$. This striking finding provides evidence for the presence of strong spin-orbit coupling (SOC) in the NbN superconductor. Without this SOC, CAR cannot occur in the spin-polarized edges due to the $s$-wave superconducting pairing in NbN. Our Bogoliubov–de Gennes spectrum, calculated for integer $1 \leq \nu \leq 6$, supports this observation [Figs. 7 and 17; see also Fig. 2(b), inset, Appendix B, and Supplemental Material for the theory model [35]]. Figures 2(c) and 2(d) show the energy spectrum of $\nu = 1$ and 2 without and with SOC, represented in our Hamiltonian by the term $j_{KR}A$. A pairing gap $\Delta_{\text{ind}}$ does not open without SOC [Fig. 2(e)]. The inclusion of SOC allows for pairing ($\Delta_{\text{ind}} > 0$) between the two counterpropagating spin-up edges of $\nu = 1$, and separately between the two additional counterpropagating spin-down edges of $\nu = 2$ [Fig. 2(d)]—pairing between the edges with opposite spin polarization is forbidden owing to the difference in their Fermi wave vector $k_F$. Interestingly, our comparable $p_{\text{CAR}}$ for all integer $\nu$ and its magnetic field insensitivity for small $I_{\text{ex}}$ and $T$ (compared to $\Delta_{\text{ind}}$) is consistent with a topologically nontrivial $\Delta_{\text{ind}}$.

Our $p_{\text{CAR}}$ remains much smaller than unity [36] because of dissipation (subgap transport due to vortices expected for our type-II superconductor) as well as direct tunneling of incoming charges to outgoing edge modes without Andreev reflection. Direct tunneling is microscopically related to disorder in the graphene-superconductor interface, which we effectively model with disordered pairing [37] (see CAR resistance in Supplemental Material [35] and Figs. 18 and 19). Our model does not include edge reconstruction [38] or charge accumulation at the superconductor interface (both discussed below) but accounts for Andreev edge states demonstrated in quantum Hall–superconductor hybrids without counterpropagating edge modes [16,18,19,30,39,40]. Andreev edge states do not result in a pairing gap (Fig. 17) and in a graphene-based device produce a response sensitive to small changes in gate voltage, magnetic field, and the length of the superconductor interface, which averages to zero [18]. In contrast, our CAR response is robust, consistently negative, and independent of the interface length (Fig. 6), suggesting an induced pairing. We note that Ref. [19] finds consistently negative CAR response in InAs-based devices in a geometry excluding counterpropagating edge modes and supporting Andreev edge states.

A potential link between CAR and the induced pairing may also be revealed by spectroscopy. We have varied the energy of the injected charges by tuning $V$, serving as the bias voltage [Fig. 1(b)], and found CAR to be limited to low energies [Figs. 2(c) and 2(f)]. Notably, the energy range of CAR in $\nu = 2$ is larger than in $\nu = 6$, consistent with a larger pairing gap for lower fillings owing to their edge modes’ closer proximity to the superconductor (Fig. 19). This gate dependence excludes, for integer fillings, the effects of nonproximitized residual edge modes originating from the work function mismatch between the graphene channel and the superconductor, which would result in $\nu$-independent spectroscopic properties. (A work function mismatch tends to produce a heavily doped region near the contact [41], whose precise physical extent in our device cannot be measured directly.)

### IV. MAGNETIC FIELD DEPENDENCE

We now extend our analysis of $p_{\text{CAR}}$ to fractional $\nu$ and a larger $B$ range. Figure 3(a) shows $p_{\text{CAR}}$ and the accompanying $\nu R_{XX}$ ($R_{XX}$ normalized for different $\nu$), with the measured $R_{\text{CAR}}$ and $R_{XX}$ at $B = 9$ T plotted in Fig. 3(b). For all fractional $\nu$ with $R_{\text{CAR}} < 0$, we find a negligibly small $R_{XX}$ which excludes bulk conduction. Bulk conduction does, however, result in $R_{XX} > 0$ observed for lower values of $B$ or for fractional $\nu$ with smaller excitation gaps. Figure 3(c) shows the $B$ dependence of $p_{\text{CAR}}$ for several fractional and integer $\nu$. In this $B$ range, we again find for several fillings a $p_{\text{CAR}}$ that does not depend on $B$ and is comparable between different $\nu$, this time including several fractional $\nu$ [Fig. 3(b), inset]. Importantly, this observation is contrasted by particlelike fillings $\nu = 1/3$ and 4/3, which present a well-developed FQH state (negligible $R_{XX}$) together with a strong magnetic field dependence. The origin of this behavior is presently unknown and can be either a result of edge reconstruction or intrinsic to the superconducting pairing of FQH edges. Identifying these scenarios theoretically is challenging due to the interacting many-body nature of a FQH state not allowing for our Bogoliubov–Dirac-Landau analysis. Experimentally, we find a $p_{\text{CAR}}$ that strongly depends on $B$ and $T$ (presented below) only for the particlelike FQH states whose $p_{\text{CAR}}$ is significantly larger than that for integer $\nu$. Such $B$ and $T$ dependences are much weaker for $\nu = 2/3$ and $5/3$, as well as for the FQH states in our second device—although with a slightly different geometry and measured at higher $T > 1.6$ K—whose $p_{\text{CAR}}$ are comparable to those for integer states (Figs. 4,8–14).
V. GRAPHENE REGION SURROUNDING THE SUPERCONDUCTOR

To investigate the pronounced CAR of particlelike fractions, we simultaneously control top and bottom gate voltage in our second device, which allows for tuning the density in the ∼100 nm narrow graphene region surrounding the superconductor without affecting the bulk filling $\nu$ [Fig. 4(a)]. This experimental knob affects the filling $\nu'$ around the superconductor, which enables the probing of the profiles of the FQH edges and the extent of charge accumulation at the superconductor interface (contact-induced doping). Varying $\nu'$ for constant $\nu$ changes the existing edge modes along the superconductor, as shown in Fig. 4(b) for the depleted, neutral ($\nu'$ matches $\nu$) and accumulated narrow graphene region. First, we observe that the particlelike fractional states $\nu = 1/3, 2/5$ [Fig. 4(c)] produce a qualitatively different behavior from the integer [Fig. 4(d)] or hole-conjugate fractional states $\nu = 3/5, 2/3$ [Fig. 4(e)]: while CAR in the particlelike fractions is limited to the matched regime, CAR in the integers or hole-conjugate fractions is not suppressed in the accumulation regime. This could be related to the details of edge profile and reconstruction in, for instance, $\nu = 1/3$ and $2/3$ which equilibrate [42,43]. Second, for all FQH states a small decrease of $\nu'$ compared to $\nu$ suppresses CAR. These observations are compatible with the scenario that the accumulation region in the graphene channel related to contact-induced doping is very narrow (∼100 nm) and thus fully proximitized, which can potentially explain the strong CAR response for particlelike FQH edges. However, we cannot rule out alternative explanations such as a mundane disorder-induced origin for our pronounced CAR in certain fractions.
FIG. 4. Graphene region surrounding the superconductor. (a) Inset illustrates the region around the superconductor in our devices. The \( \sim 100 \) nm narrow graphene region surrounding the superconductor is not covered by top graphite (gate). This enables tuning the density (filling \( \nu \)) around the superconductor without affecting the bulk filling \( \nu \). Here, \( \nu \) is set simultaneously by top and bottom gate: \( \nu \propto (V_T + c \cdot V_B) \) where \( V_T \) is the top gate voltage, \( V_B \) the bottom, and \( c = 0.51 \) the ratio of the capacitive couplings of top and bottom gate to the bulk graphene. In contrast, graphene surrounding the superconductor is capacitively coupled primarily to the bottom gate (\( \nu \propto V_B \)). Color plot shows CAR for several fillings \( \nu \) as a function of \( V_B \). The symbols mark the edge mode configurations illustrated in (b) corresponding to the matched (\( \nu = \nu' \)) regime for which \( V_T = 0 \). Increasing \( V_B \) results in the accumulation (\( \nu < \nu' \)) regime, whereas decreasing \( V_B \) results in the depletion (\( \nu > \nu' \)) regime. The dashed line marks \( V_B^0 \), the value of \( V_B \) for which \( V_T = 0 \). (b) Illustrations of the edge mode configuration of the particlelike \( \nu = 1/3 \), the hole conjugate \( \nu = 2/3 \), and the integer \( \nu = 1 \) for varying \( \nu' \). The dashed and the solid green line surrounding the superconductor illustrate integer edge modes potentially induced by an increased charge carrier density at the superconductor interface (contact-induced doping). The thin dashed arcs between the two edge modes \( 1/3 \) (orange line) and \( 1 \) (green line) depict their equilibration with each other. CAR is absent in the depletion regime, present in the matched regime. Our observed suppression of CAR in \( \nu = 1/3 \) for \( \nu' = 1 \) suggests an incomplete equilibration between the edge modes 1/3 and 1 along the electrostatically defined edge (accumulation, top panel). In contrast, CAR in \( \nu = 2/3 \) is present for \( \nu' = 1 \), which is consistent with a complete equilibration between the two edge modes along the physical sample edge (accumulation, middle panel). Shaded area in the bottom right accumulation panel depicts a compressible region. The presence of CAR for this filling configuration (\( \nu = 1/3 < \nu' \)) suggests negligible tunneling between the edge mode and the compressible region. We also note that we did not observe oscillatory conductance between the two fillings 1/3 and 1, presumably due to the 1D nature of the interface between them and higher temperatures than in the recent experiment [44]. (c) CAR in the particlelike fractional states is limited to small \( |V_B - V_B^0| \). (d),(e) CAR in the integer and the hole-conjugate fractional states are not suppressed for large \( V_B - V_B^0 \).
VI. TRANSPORT SPECTROSCOPY AND TEMPERATURE DEPENDENCE

Next, we perform spectroscopy in fractional $\nu$ by varying $V$ and monitoring $p_{\text{CAR}}$ and $R_{xx}$ simultaneously at different temperatures. Figures 5(a)–5(d) show CAR for $\nu = 1/3$ and 2/5, both limited to an energy range below $|eV| \sim 1$ meV and to low $T$ (see also the color plots as insets and Fig. 15, which shows $R_{\text{CAR}}$ instead of $p_{\text{CAR}}$). Increasing $V$ and $T$ bring the injected charges above the excitation gap of the fractional fillings (bulk conduction) or above $\Delta_{\text{ind}}$ (Bogoliubov-quasiparticle transport without Andreev reflection), both suppressing CAR. The comparison of fractional $\nu$ (excitation gaps in the same range as $\Delta_{\text{ind}}$) with $\nu = 2$ (largest Landau gap, significantly larger than $\Delta_{\text{ind}}$) suggests that for our fractional $\nu$ an increasing $V$ suppresses CAR primarily due to bulk conduction, while the suppression with $T$ is due to both bulk conduction and Bogoliubov-quasiparticle transport [Figs. 5(a) and 5(c) versus Figs. 5(a), left-hand inset; Figs. 5(b) and 5(d) versus Fig. 14].

We proceed with the temperature dependence of $p_{\text{CAR}}$ for several integer and fractional $\nu$ [Fig. 5(e)]. We find consistent CAR below $\sim 5$ K for all well-developed fillings, demonstrating the robustness of superconductivity in the FQH state. Above this $T$ increasing quasiparticle transport overcomes CAR. The CAR at $\nu = 1$ and 2 persists up to a larger $T$ in our second device measured using a different setup (Fig. 14), indicating that quasiparticle transport can be further decreased.

Figure 5(f) shows the vertical cuts from Fig. 5(e) for several $\nu$. We find a $p_{\text{CAR}}$ saturating at low temperatures for the integer fillings $\nu = 1$, 2 as well as $\nu = 2/3$. Interestingly, a topological $\Delta_{\text{ind}}$ is also expected to provide a temperature-independent behavior for integer $\nu$, equivalent to the $B$ independence presented in Figs. 2 and 3. In stark contrast, the particlelike fillings $\nu = 1/3$ and 2/5 show a clear temperature dependence down to the lowest $T$ (Fig. 16), with the $p_{\text{CAR}}$ of $\nu = 1/3$ reaching above 6% at $T = 15$ mK, as shown in Fig. 3 ($B = 9$ T).

An increasing $p_{\text{CAR}}$ with decreasing $T$ could be a feature of the superconducting pairing of fractional charges $e^*$ [Fig. 5(f), right-hand inset, and Fig. 20]. Here, considering $\nu = 1/3$, CAR converts an incoming electronlike $e^* = 1/3$ to an outgoing holelike $-e^* = -1/3$ adding a 2/3 charge to the superconductor. Such $T$ dependence is not expected for pairing of integer charges in FQH edges [Fig. 5(f), left-hand inset], where, for $\nu = 1/3$, three incoming $e^*$ bunch together, which are then converted to three bunched $-e^*$ leaving the superconductor. In this case, CAR vanishes ($R_{\text{CAR}} = 0$) at zero temperature, which is not observed for both our devices at our lowest $T$. Although our experiments do not constitute direct evidence for pairing of fractional charges, which can be complicated by contact-induced doping, our $p_{\text{CAR}}$ for fully developed particlelike FQH states being larger than that of integer and hole-conjugate fractional $\nu$ suggests a different underlying mechanism for pronounced fractional CAR. Direct evidence for fractional charge pairing could be obtained in measurements which are sensitive to the charge of the Andreev-reflected particles. Additionally, strong bulk disorder in the NbN superconductor corresponding to a mean free path $<1$ nm, which is smaller than the magnetic lengths $\sim 10$ nm in our $B$ range, is expected to result in momentum-nonconserving processes. While such processes should provide an additional $s$-wave pairing for fillings $\nu \geq 2$, our experiments did not resolve an $s$-wave contribution which would result in different magnetic field dependence for $\nu = 1$ versus larger fillings $\nu \geq 2$. This is surprising because such strong disorder strengths are expected to suppress $p$-wave pairing and leave $s$-wave pairing as the sole mechanism responsible for superconductivity.

Our presented experiments show Andreev reflection in the fractional quantum Hall state, enabled by spin-orbit coupling in the superconductor. Following experiments, including tunneling [29,47], noise [48,49], and supercurrent [50] measurements, will be able to reveal direct evidence for pairing of fractional charges in this hybrid system. Accessing the topological properties will require significant increase of CAR probability by a suppression of edge disorder and vortices, both challenging to be theoretically accounted for in order to predict the experiment, such as temperature dependence.
FIG. 5. Transport spectroscopy and temperature dependence. (a) $p_{\text{CAR}}$ at $\nu = 1/3$ as a function of the incoming edge mode potential $V$ (excitation) for different temperatures $T$. Crossed Andreev reflection is limited to below $|eV| \sim 1$ meV and $T < 6$ K. Increasing excitation and $T$ suppress CAR. Left-hand inset of (a) shows $p_{\text{CAR}}$ at $\nu = 2$ with a $V$ and $T$ dependence similar to that at $\nu = 1/3$ apart from a larger $V$ range and a smaller $p_{\text{CAR}}$. Right-hand inset of (a) shows the $p_{\text{CAR}}$ corresponding to (a), divided by the incoming edge mode potential $R_{\text{CAR}} + R_{\text{XY}}$, allowing a direct comparison to $p_{\text{CAR}}$. Increasing excitation and $T$ result in bulk conductance which suppresses CAR. Inset of (b) shows $R_{\text{XX}}$ corresponding to the right-hand inset of (a). (c), (d) Same as (a) and (b) but for $\nu = 2/5$, which shows a $V$ and $T$ dependence similar to that for $\nu = 1/3$. (e) $p_{\text{CAR}}$ as a function of $\nu$ for varying $T$. $R_{\text{CAR}} < 0$ below $\sim 5$ K for all well-developed fillings (highlighted with arrows). (f) Vertical cuts from (e). At $\nu = 2/3$, 1, and 2, $p_{\text{CAR}}$ saturates below $\sim 4$ K with decreasing $T$, whereas at $\nu = 1/3$ and 2/5 continues to increase in amplitude without saturating. The shades are the uncertainty in the measurement while sweeping the gate voltage. The uncertainty is larger for fillings represented by fewer data points. Insets of (f) illustrate two different mechanisms of charge transport to the superconductor. Left-hand schematic illustrates bunching of fractional charges of $+e/3$ to form integer charges of $e$ that pair. This mechanism converts three incoming $e^*$ to an outgoing $-e$, an integer-charged hole, adding $2e$ to the superconductor. Right-hand schematic illustrates the pairing of fractional charges, a mechanism that converts $e^*$ to $-e^*$, adding $2e/3$ to a fractional topological superconductor. For pairing of integer charges, $p_{\text{CAR}}$ vanishes at $T = 0$. In contrast, $p_{\text{CAR}}$ monotonically increases in amplitude with decreasing $T$ when fractional charges pair.
FIG. 6. The presented devices. Data presented in Figs. 1–3, 5, 15, 16 are taken from device 1; Figs. 4, 8–14 are taken on device 2. The superconductor in device 1 is ~1 μm long resulting in a ~2-μm-long graphene-superconductor interface. The superconductor in device 2 is ~2 μm long, which results in a ~4-μm-long interface. No systematic dependence of $p_{\text{CAR}}$ on the interface length has been observed. The width of the superconductor is <100 nm in both devices.

FIG. 7. Evolution of Bogoliubov–de Gennes spectrum when including superconductivity without spin-orbit coupling. (a) Illustration of the edges separated by vacuum for $\nu = 1, 2$, and their calculated Bogoliubov–de Gennes spectrum. Momentum $k$ in units of $l_B^{-1}$ with $l_B$ the magnetic length. $k$ to $-k$ symmetry is a result of physical ($Z_2$) symmetry—a reflection with respect to the separating vacuum [or superconductor for (b)], which exchanges the left mover and the right mover. Spectrum is doubled to show both electronlike (solid lines) and holelike (dashed lines) excitations. Color code indicates the spin and the direction of propagation. (b) Inclusion of superconductivity ($\Delta > 0$) without spin-orbit coupling ($\lambda_R = 0$) does not affect the spin polarization. This leaves the zero-energy crossings spin polarized, which cannot be gapped by an s-wave superconductor—an induced pairing gap $\Delta_{\text{int}}$ does not open. (b) Same as Fig. 2(c). For these simulations and the ones presented in Fig. 2, the system parameters are set as follows: $\Delta_1 = 0.3 \varepsilon_0$, $\Delta_2 = 0.2 \varepsilon_0$ ($\Delta_{1,2} = 0$ for vacuum), $m_s = 3 \varepsilon_0$, $m_n = 0.06 \varepsilon_0$, $\mu_s = 8 \varepsilon_0$ ($\mu_s = 0$ for vacuum), $\mu_n = 0.2 \varepsilon_0$ (for $\nu = 1$), $\mu_n = 0.45 \varepsilon_0$ (for $\nu = 2$), $g_R = 0.2 \varepsilon_0$, $\lambda_{RS} = \lambda_{S0} = 0$ and $\lambda_{R^*} = 0.2 \varepsilon_0$ ($\lambda_{R^*} = 0$ for no spin orbit), $W_s = 6l_B = 1.7 \xi_0 = 62.7$ nm, $\varepsilon_0 = 89$ meV.
FIG. 8. Crossed Andreev reflection in the FQH state in device 2. (a),(b) $R_{\text{CAR}}$ and $R_{\text{XX}}$ as a function of gate voltage at $B = 13$ T for different temperatures $T$. $R_{\text{CAR}} < 0$ at the highlighted fillings indicates crossed Andreev reflection. (c),(d) Same as (a) and (b) but for integer fillings 1 and 2 in a larger $T$ range. (e),(f) $P_{\text{CAR}}$ at $T = 1.7$ K, respectively, from (a) and (c).
FIG. 9. Spin-orbit coupling in device 2. (a) $p_{\text{CAR}}$ as a function of filling $\nu$. Integer fillings, including the spin-polarized $\nu = 1$, have a comparable $p_{\text{CAR}}$, an evidence for the pairing mechanism enabled by spin-orbit coupling. The dashed line is the average $p_{\text{CAR}}$. Inset shows $R_{\text{CAR}}$ measured at $B = 6.5, \ldots, 8.5$ T. The filling axis has been adjusted such that the measured fillings align with the integer values of the axis. This procedure to convert gate voltage to filling is present herein and in Figs. 2(a) and 2(b).

FIG. 10. Magnetic field dependence in device 2. (a) $p_{\text{CAR}}$ and $R_{XX}$ (normalized for different $\nu$) as a function of filling and magnetic field $B$. Crossed Andreev reflection ($R_{\text{CAR}} < 0$) is seen for all well-developed $\nu$. Bulk conduction suppresses CAR observed for lower values of $B$ or for fractional $\nu$ with smaller excitation gaps. (b) $R_{\text{CAR}}$ and $R_{XX}$ at $B = 13$ T from (a). Bulk conductance ($R_{XX}$) is negligible for all highlighted fillings except $\nu = 2/3$, a complication related to equilibration in the contact, which is limited to this measurement.
FIG. 11. Transport spectroscopy in device 2 (part 1). (a)–(f) $R_{\text{CAR}}$ and $R_{XX}$ at several fillings as a function of the incoming edge mode potential $V$ (excitation) for different temperatures $T$. Crossed Andreev reflection ($R_{\text{CAR}} < 0$) is limited to below $|eV| \sim 1$ meV. Increasing excitation and $T$ suppress CAR by increasing bulk conductance for these fillings in this device. The features in (e) and (f) appearing at $V \sim 1$ mV correspond to the magnon excitation [45,46], missing in other integer and fractional quantum Hall states.
FIG. 12. Transport spectroscopy in device 2 (part 2). (a)–(f) $P_{\text{CAR}}$ and $R_{\text{XX}}$ at several fillings as a function of the incoming edge mode potential $V$ (excitation) for different temperatures $T$. Crossed Andreev reflection ($R_{\text{CAR}} < 0$) is limited to below $|eV| \sim 1$ meV, $T \sim 4$ K for $\nu = 1/3$ and $2/3$, and $T \sim 6$ K for $\nu = 1$. 
FIG. 13. Temperature dependence in device 2 (part 1, fractional fillings). (a),(b) $P_{\text{CAR}}$ and $R_{XX}$ (normalized for different $\nu$) as a function of filling for varying $T$. Crossed Andreev reflection ($P_{\text{CAR}} > 0$) is seen for the well-developed fillings $\nu = 1/3, 2/3$, and 1, as well as the fillings with smaller excitation gaps $\nu = 2/5$ and $3/5$, all highlighted with arrows. (c),(d) Vertical cuts, respectively, from (a) and (b). The shades represent the standard deviation.

FIG. 14. Temperature dependence in device 2 (part 2, integer fillings). (a),(b) $P_{\text{CAR}}$ and $R_{XX}$ (normalized for different $\nu$) as a function of filling for varying $T$. Crossed Andreev reflection ($P_{\text{CAR}} > 0$) is seen for both integer fillings $\nu = 1$ and 2. (c),(d) Vertical cuts, respectively, from (a) and (b). The shades represent the standard deviation. Crossed Andreev reflection at $\nu = 2$ is undisturbed by bulk conductance and suppressed when the NbN superconductor turns normal at $T \sim 9$ K.
FIG. 15. Transport spectroscopy in device 1 showing $R_{\text{CAR}}$ and $R_{XX}$. (a),(b) $R_{\text{CAR}}$ and $R_{XX}$ for $\nu = 1/3$ as a function of the incoming edge mode potential $V$ (excitation) for different temperatures $T$. Crossed Andreev reflection ($R_{\text{CAR}} < 0$) is limited to below $|eV| \sim 1$ meV. (c),(d) Same as (a) and (b) but for $\nu = 2/5$, which shows a $V$ and $T$ dependence similar to that for $\nu = 1/3$.

FIG. 16. Temperature dependence of crossed Andreev reflection at $\nu = 1/3$ and $2/5$ in device 1. (a) shows the CAR probability at $\nu = 1/3$, (b) shows that at $\nu = 2/5$. (c) and (d) respectively show $R_{\text{CAR}}$ at $\nu = 1/3$ and $2/5$ together with the corresponding $R_{XX}$. Probability of crossed Andreev reflection in fillings $\nu = 1/3$ and $2/5$ is rapidly increasing with decreasing $T$ down to the lowest temperature, at which bulk conductance vanishes ($R_{XX} \sim 0$). Data extracted from Figs. 5(a)–5(d).
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Ö. G., Y. R., and P. K. conceived the idea and designed the project. P. K. supervised the project. Ö. G., Y. R., S. Y. L., and J. Z. fabricated the devices. H. S. and A. V. set up the theoretical model. Y. H. L. and A. Y. consulted on and helped in different stages of the fabrication process and analysis. Ö. G., Y. R., and J. Z. performed the measurements. Ö. G., Y. R., H. S., J. Z., A. Y., and P. K. wrote the paper with input from all authors.

APPENDIX A: EXPERIMENTAL METHODS

1. Assembly of the heterostructure

We assembled our five-layer graphite–hexagonal boron nitride (hBN)–single layer graphene–hBN–graphite van der Waals heterostructures with the standard dry transfer technique [51], using a polycarbonate- (PC) polydimethylsiloxane (PDMS) stamp. We exfoliated the flakes via thermal release tape onto (doped) Si substrates covered with 285-nm-thick SiO$_2$—the same substrate on which the devices were fabricated. To increase the size of the flakes we baked the substrates, the tape carrying bulk hBN and graphite still adhered, at 100 °C for 1 min on a hot plate before releasing the tape. After exfoliation we annealed the substrates with the flakes at high vacuum (~3 × 10$^{-7}$ mbar) and 350 °C for 20 min to remove the tape residues, ramping the temperature from 20 °C over 3 h. We did not perform these two heat treatments on single-layer graphene to avoid modifying its intrinsic properties. We determined the cleanness and thickness of the flakes by first optical and then atomic force microscopy. The graphite layers in our heterostructures were determined to be ~1.5–3 nm thick (5–10 layers) while the hBN layers were ~50 nm (top) and 90–100 nm (bottom), all confirmed to be atomically flat. We used a thicker bottom hBN to prevent an electrical connection between the bottom graphite and the overlapping superconductor (see Appendix A2). After choosing suitable flakes, we assembled the heterostructure in a glovebox (H$_2$O < 0.1 ppm, O$_2$ < 0.1 ppm) to decrease contamination between the layers.

To make the stamp, we prepared a PC solution (8 wt %) and pipetted it onto a glass slide. We placed another glass slide on top of the solution and left the resulting film to cure in ambient conditions. We then placed a rectangular block (~8 × 5 mm) of PDMS (Gel-Pak) on a separate glass slide and transferred the PC film (with a larger area than that of the PDMS) on top. To ensure adhesion between the PC film and the glass slide we baked the stamp at 180 °C for 5 min, and then mounted the finished stamp on a dry transfer setup.

Before transferring flakes, we flattened the surface of the PC film by touching the stamp onto a bare substrate held by vacuum on a sample stage at 155 °C. After a cooldown period during which the PC film detached from the bare substrate, we replaced the substrate with the one containing the top graphite. The stamp was used to pick up the graphite at ~130°C, followed by a cooldown until the stamp detached. We then used this top graphite to pick up the top hBN via van der Waals force. In this step, the top graphite approached the hBN extremely slowly at 150°C–155°C to minimize the formation of bubbles. We picked up this hBN, the graphene, and the bottom hBN with the same procedure described above. We then dropped the whole stack (graphite–hBN–graphene–hBN) from the stamp onto the bottom graphite at ~170°C. Importantly, dropping the stack at this high temperature allowed us to push out the bubbles formed between the layers during assembly [52]. Finally, we released the PC film at 190 °C, ending the transfer process. We completed the assembly by removing polymer residues from the heterostructure, first leaving the chip in chloroform overnight and then annealing the heterostructure at high vacuum (~3 × 10$^{-7}$ mbar) and 350 °C for 30 min after a 3 h temperature ramp.

2. Nanofabrication

We defined the superconductor, normal leads, and the shape of the device via electron beam lithography followed by reactive ion etching (RIE). For RIE, we used a CHF$_3$, Ar, and O$_2$ gas mixture, whereas we excluded CHF$_3$ for the selective removal of the top graphite. Both the superconductor and the normal leads contact the graphene from its edge, and were deposited following RIE in one lithography step in which the etch mask also served as the deposition mask. For the superconductor, we first
selectively etched the top graphite that would otherwise surround the superconductor, leaving a ~100 nm separation to avoid an electrical connection. In the subsequent lithography step, we deposited the superconductor after vertically etching the heterostructure beyond the graphene layer using an etch stop. This etch stop is based on the \textit{ex situ} measured conductance of a test area of identical layer composition and leaves most of the bottom hBN unetched which insulates the superconductor from the bottom graphite. Unlike the superconductor, the normal leads do not overlap the top or bottom graphite, and instead connect to the fractional quantum Hall heterostructure through graphene that extends beyond both graphite layers (Fig. 6).

The normal leads are Cr/Pd/Au (2/7/150 nm), thermally evaporated on a rotating sample stage with ~15° tilt. We deposited our superconductor in an AJA International UHV hybrid system (base pressure ~10⁻⁷ Torr). The superconductor deposition started with electron beam evaporation of Ti (10 nm, with rotation and 15° tilt) immediately followed by dc magnetron sputtering of Nb/NbN (5/75 nm, without tilt) at a pressure of 3 mTorr and a power of 200 W using a Nb target. We deposited Nb in an Ar environment. For NbN we used an Ar/N₂ (50/6 sccm) gas mixture. This superconductor has a critical temperature Tc ∼ 12 K at B = 0 T. An overview of the fabricated devices is given in Supplemental Material, Table S4.

3. Measurement

The parts of the graphene that extend beyond the top and bottom graphite, which connect the normal leads to the fractional quantum Hall heterostructure, were doped by using the substrate as a global gate. The top and bottom graphite layers were used as gates to control the charge carrier density in the fractional quantum Hall heterostructure. Device 1 used top graphite as the gate while the bottom graphite was grounded, whereas the opposite was the case for device 2 except in Fig. 4, where device 2 used top and bottom graphite simultaneously as gates.

We measured device 1 in a variable temperature inset (VTI) at T ≥ 1.75 K and in a dilution refrigerator with its cold finger at T = 15 mK, and device 2 in a different VTI at T ≥ 1.6 K. For the measurements in VTIs, we used a RC filter attached to the chip carrier, whereas the dilution refrigerator was equipped with RC, copper powder, and pi filters (LCL filter manufactured by Mini-Circuits) all thermalized to the cold finger. We reproduced our observations after every thermocycle for both our devices. Importantly, however, we were unable to measure crossed Andreev reflection without a filter.

We used the standard ac lock-in technique (f = 17.77 Hz) with an excitation current I_{exc} = 5 or 10 nA for the VTI measurements and I_{exc} = 1 or 5 nA for the dilution refrigerator measurements (ac circuit simulations of the device within the measurement circuit are provided in Refs. [35,53]). All presented measurements used a single source and the superconductor as the single drain. Because of the finite resistance (wiring and filters) between the superconductor and the breakout box at room temperature, V_{CAR} and V [Fig. 1(b)] were measured relative to the superconductor potential. Our superconductor coupled to the fractional quantum Hall edges branches out to four separate leads (Fig. 6). Two branch out immediately after leaving the heterostructure (one used as the drain, the other to monitor the potential), and the remaining two (left floating) after a narrow strip with identical dimensions as the one coupled to the fractional quantum Hall edges. This strip allowed for an independent test of our superconductor. An overview of the measured devices is given in Supplemental Material, Table S5 [35]. The data that support the findings of this study are available in Ref. [53].

APPENDIX B: THEORY

The inset of Fig. 2(b) (see also Fig. S1 in Supplemental Material [35]) shows the schematic of the vertically shrunk experimental system which satisfies I_{exc} = v · e^2/h · (V − V_{CAR}). A p_{CAR} = 1 implies R_{CAR}/R_{XY} = −1/2. In Supplemental Material [35], we present a low-energy theory of graphene in the presence of a strong out-of-plane B [54] (Fig. S2 in Supplemental Material) and compute crossed Andreev reflection for integer fillings. In the following, we first highlight the results of our integer quantum Hall theory analysis supporting our experimental findings, and then renormalization flow equations which describe CAR response in the FQH edge as the temperature is varied.

1. CAR in the integer quantum Hall state

We consider counterpropagating edge modes along either side of the superconductor [Fig. 2(b), inset, y axis is along the superconductor]. For reference, we show in Fig. S4 the armchair edge energy spectra in the absence and presence of the Dirac mass term, where we replace the superconducting region with vacuum (see Supplemental Material for details [35]). Without any (valley) symmetry breaking, the quantum Hall sequence is ν = 2, 4, 6, .... Adding the Dirac mass term modifies the sequence into ν = 1, 2, 4, ... with a spin and valley polarized ν = 1. Turning on superconductivity yields Fig. 17, with the top panels for ν = 1 and the bottom panels for ν = 2. For any integer filling zero-energy band crossings are between particle-hole partners with identical spin polarization. Therefore, there is no direct mechanism for s-wave pairing (even for spin-unpolarized fillings such as ν = 2). However, the large spin-orbit coupling in NbN superconductor provides a necessary ingredient for a spin-flip process allowing for a pairing between electrons with the same spin polarization. We account for spin-orbit coupling via [55]

$$\hbar_{SO} = \lambda_{SO} \tau_z s_x \sigma_z + \lambda_{R,x} \tau_z s_x \sigma_x − \lambda_{R,y} s_y \sigma_y.$$  

Because the induced pairing between the counterpropagating edge modes exponentially decays as a function of the...
superconductor thickness, a gap does not open for a thick superconductor (left-hand panels). This is the experimental scenario in which Andreev edge states govern the transport \[16,18,19,30,39,40\]. Reducing the thickness of the superconductor hybridizes the edge modes along both sides of the superconductor and opens a gap in the Bogoliubov–de Gennes spectrum (middle panels). The band crossings and the resulting hybridization occur for each mode separately—each edge mode (for instance, spin-up and spin-down) experiences a similar induced pairing (gap opening) mechanism. The gap opening term is a spin-polarized pairing of the form \(\psi_L \sigma \psi_R \), which originates from the combination of the spin-singlet pairing of the parent superconductor and spin-orbit coupling. Turning off spin-orbit coupling results in zero-energy band crossings remaining intact (right-hand panels). The degeneracy points (band crossings at finite energies) near \(k_y = 0\) are lifted for a thin superconductor regardless of the presence of spin-orbit coupling. This is because those degeneracies open by either spin-singlet intraedge pairing or simply direct tunneling. Nevertheless, these processes do not play any role for the zero-energy band crossings between two spin-polarized modes since the former process does not open a gap in a spin-polarized channel and the latter is forbidden at finite \(k\) due to violating momentum conservation. This implies that \(\nu = 2\) edge modes can be treated effectively as two copies of \(\nu = 1\).

Our numerical analysis confirms that the induced gaps within different edge modes are of the same order and decrease as the superconductor is made thicker (Fig. S5 in Supplemental Material [35]). To study crossed Andreev reflection in a realistic experimental system we use an effective edge theory (see Supplemental Material for details [35]). We consider a disordered pairing potential,

\[
V_{\text{dis}} = \Delta_1(y) \tau_y \sigma_y + \Delta_2(y) \tau_y \sigma_y,
\]

which we vary over the minimum coherence length \(\xi_0 \sim v_F/\Delta_m\). Here, both \(\Delta_1\) and \(\Delta_2\) are random variables drawn from a uniform distribution \([-\Delta_m, \Delta_m]\). (Note that here we suppress the complex phase of the pairing potential for simplicity. Our numerical results for disorder simulation do not exhibit qualitative change when we choose a complex pairing potential.) The resulting CAR is shown in Fig. 18. It is evident that in both the clean and the disordered case the zero-bias \(R_{\text{CAR}}\) is inversely proportional...
to the filling $\nu$, consistent with the experiment. Notably, the experimental observation of CAR vanishing at finite energies can only be recovered in our calculation by introducing disorder which breaks translation symmetry and effectively produces direct tunneling terms. For the clean case, CAR remains consistently finite even at high energies. This implies that the pairing gap and disorder together determine the energy range of CAR. Finally, Fig. 19 compares the calculated CAR response with the experiment, qualitatively reproducing the observed...
difference between $\nu = 2$ and 6. The larger energy range of CAR in $\nu = 2$ is due to its counterpropagating edge modes being closer to the superconductor than those of $\nu = 6$, which results in a stronger hybridization (larger $\Delta_{\text{int}}$). Apart from enabling direct tunneling, translation symmetry breaking also enables gap-opening superconducting pairing terms between counterpropagating edge modes of different Landau levels (for example, in $\nu = 2$ the inner edge mode can now pair with the counterpropagating outer edge mode), which is not possible for a disorder-free graphene-superconductor interface. This mechanism provides an additional $s$-wave contribution for fillings $\nu \geq 2$. (Note that for $\nu = 1$ no such $s$-wave term is present even with disorder.) However, such momentum-violating pairing terms hybridizing counterpropagating edge modes from different Landau levels are likely not dominant in the experimental system because no strong filling dependence is observed, particularly for $\nu = 1$ versus larger fillings $\nu \geq 2$.

2. CAR in the fractional quantum Hall state

Now we turn to the FQH state, which requires different approaches from the integer QH CAR discussed above. For simplicity, we shall consider only the $\nu = 1/m$ Laughlin states (with odd $m$), where the edge theory is single component. The system is described by two chiral edge modes near top and bottom of the sample:

$$\mathcal{L}_{\text{edge}} = \frac{1}{4\pi v} \int dy [ \partial_y \phi_L \partial_y \phi_L - V \partial_y \phi_L \partial_y \phi_L ]$$

$$+ \frac{1}{4\pi v} \int dy [ - \partial_y \phi_R \partial_y \phi_R - V \partial_y \phi_R \partial_y \phi_R ],$$

where

$$[\phi_{L,R}(y_1), \phi_{L,R}(y_2)] = \mp \frac{i \pi}{m} \text{sgn}(y_1 - y_2),$$

$$[\phi_{L}(y_1), \phi_{R}(y_2)] = i \frac{\pi}{m}.$$

$\phi_{L,R}$ are chiral bosons [56,57]. Using this formulation, the electric charge density associated with $\phi_0$ is given by

$$\rho_a = \frac{1}{2\pi} \partial_a \phi_0,$$

and the $i$th electron operator is described by the vertex operator,

$$\Psi_{e,L/R} = e^{i\nu^{-1} \phi_{a,L/R}}.$$

In the right half of the system [recall Fig. 2(b), inset] the two edge modes are decoupled. In the left half, the two edge modes are coupled via the following induced pairing term:

$$\mathcal{L}_{\text{pairing}} = \Delta \int_0^\infty dx \cos \nu^{-1}(\phi_L + \phi_R).$$

Note that we drop the Klein factor since it commutes with other terms in the Hamiltonian. The bare scaling dimension of this term is $1 - \nu^{-1}$. However, we work in the strong coupling limit $\Delta \to \infty$. The ground state of the $x > 0$ region is obtained by pinning the field $\nu^{-1}(\phi_L + \phi_R) = 2\pi n$, where $n = 0, 1, 2, \ldots, \nu^{-1} - 1$. This in turn implies not only $\phi_L = \phi_R = 0$ but also $e^{i\nu(\phi_L + \phi_R)} \neq 0$; that is, we get a condensate of quasiparticle and quasihole pairs. The scattering of a (quasi)particle impinging on the $x > 0$ region from the left region can be addressed by the following two processes:

1. coherent conversion of a right-moving quasielectron to a left-moving quasihole, which is described by the following term

$$\Gamma_{qp} \int dy \delta(y - 0^{-}) e^{i\nu(\phi_L + \phi_R)} + \text{H.c.}$$

2. coherent conversion of a right-moving electron to a left-moving hole, which is described by the following term:

$$\Gamma_{eh} \int dy \delta(y - 0^{-}) e^{i\nu^{-1}(\phi_L + \phi_R)} + \text{H.c.}$$

The latter process requires electron bunching before going through the superconductor. The scaling dimensions of the two processes are given by $\nu$ and $\nu^{-1}$, respectively. Hence, the tunneling amplitudes obey the renormalization flow equations,

$$\frac{d\Gamma_{qp}}{dl} = (1 - \nu),$$

$$\frac{d\Gamma_{eh}}{dl} = (1 - \nu^{-1}),$$

which implies that in the infrared limit (low energy), for $\nu < 1$, $\Gamma_{qp}$ is relevant while $\Gamma_{eh}$ is irrelevant. Now, we compare experimental consequences of both processes (Fig. 20).

1. $\Gamma_{qp}$: Since this term is relevant, it drives the system to a strong coupling limit fixed point $\Gamma_{qp} \to \infty$ in the low-energy limit. So, we expect that at low temperatures there is a constant CAR response proportional to $|\Gamma_{qp}|^2$ and it gradually decreases as $T^{2\nu^{-2}}$ when the temperature is increased or as $V^{2\nu^{-2}}$ when a bias voltage is applied.

2. $\Gamma_{eh}$: Since this term is irrelevant, at zero temperature the system in a weak coupling limit $\Gamma_{eh} \to 0$ and there is no CAR response. We expect that as we increase the temperature or apply a bias voltage, the CAR response increases as $T^{2\nu^{-1} - 2}$ or $V^{2\nu^{-1} - 2}$, respectively.
Given that the experimental data as a function of temperature show an initial plateau and gradual increase as $T$ is decreased, we conclude that $\Gamma_{\text{CAR}}$ is the dominant term governing the CAR response. Note that this scaling analysis provides a qualitative picture. Further quantitative discussion on FQH CAR requires many-body simulation of the FQH states.


[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevX.12.021057 for the theory model (Supplement I), ac circuit simulations of the measurement circuit (Supplement II), expected energy scales (Table S3), line traces of $R_{\text{CAR}}$ from Fig. 3(c) plotted along with their corresponding $R_{\text{AX}}$ (Fig. S10), color plots in linear color scale (Figs. S11–S15), figures which use low-pass smoothing next to their unsmoothed versions (Figs. S16–S23), data shown in Fig. 1(f) focusing on $\nu = 1/3$ (Fig. S24), and an overview of fabricated and measured devices (Tables S4 and S5).


[53] Measurement data, data processing and figure generating scripts of this study are available at 10.5281/zenodo.6586602.


