Quantum Nonlinear Optics with Polar J-Aggregates in Microcavities

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ABSTRACT: We predict that an ensemble of organic dye molecules with permanent electric dipole moments embedded in a microcavity can lead to strong optical nonlinearities at the single-photon level. The strong long-range electrostatic interaction between chromophores due to their permanent dipoles introduces the desired nonlinearity of the light–matter coupling in the microcavity. We develop a semiclassical model to obtain the absorption spectra of a weak probe field under the influence of strong exciton–photon coupling with the cavity field. Using realistic parameters, we demonstrate that a cavity field with an average photon number near unity can significantly modify the absorptive and dispersive response of the medium to a weak probe field at a different frequency. Finally, we show that the system is in the regime of cavity-induced transparency with a broad transparency window for dye dimers. We illustrate our findings using pseudoisocyanine chloride (PIC) J-aggregates in currently available optical microcavities.

SECTION: Physical Processes in Nanomaterials and Nanostructures

J-aggregates1–5 are arrays of dye molecules with large dipole moments that exhibit strong intermolecular electrostatic interaction, giving rise to collective effects in their coupling with electromagnetic fields. The specific set of linear and nonlinear optical properties of J-aggregates has stimulated a resurgence of interest in them for applications in modern photonics. A large linear absorption cross section combined with a narrow line width6 at room temperatures make J-aggregates attractive for the design of optical processing devices operating at low light levels. J-aggregates can be readily coupled to solid-state photonic7,8 and plasmonic9–11 structures, extending the conventional photonics to subdiffraction length scales.5 Illustrative examples of molecule-based photon processing structures can be found in nature, where photosynthetic organisms use molecular aggregates to collect light and deliver the photon energy on the scale of tens of nanometers.12

Moderately strong laser fields are commonly used to observe coherent optical phenomena in atomic gases and a few solid-state systems characterized by long dephasing times exceeding milliseconds at room temperature.13–15 Solid-state semiconducting materials have much shorter electronic coherence times on the order of hundreds of femtoseconds, which greatly increases the laser intensity required to induce coherent optical phenomena in free space. For instance, in order to observe electromagnetically induced transparency (EIT) using inorganic quantum dots with terahertz dephasing rates, the required control laser intensity should be on the order of tens of MW/cm2.16 The same applies for organic materials, including J-aggregates. Such high intensities can optically damage an organic medium.17 It is therefore necessary to replace the control lasers by the strong electric field per photon achievable in photonic structures5 in order to observe coherent optical response with organic matter at room temperature.

Experimental progress in the fabrication of organic optical microcavities has demonstrated the ability to strongly couple an ensemble of organic chromophores with the confined electromagnetic field of a cavity mode at room temperature,7,18–22 via the emergence of polariton modes in the cavity transmission spectra. The strong coupling of organic ensembles with plasmonic modes has also been demonstrated.5,9,23–26 Moreover, the regime of ultrastrong coupling with organic molecules is now within reach, where the light–matter interaction energy reaches a significant fraction of the associated transition frequency.27 These experimental advances enable the possibility of understanding and possibly manipulating the excited-state dynamics of molecular aggregates using a small number of photons.

In this Letter, we address the question whether collective multiexciton states in J-aggregates can be exploited for the coherent control of confined optical fields in photonic structures. In order to achieve this, we extend the nonlinear exciton equation (NEE) formalism28,29 to account for the
nonperturbative coupling of the medium to a confined optical field. The active medium under consideration consists of an ensemble of one-dimensional polar J-aggregate domains embedded in an optical microcavity, as shown in Figure 1a.

Each aggregate chain consists of a large number of polar organic dyes, such as pseudoisocyanine chloride (PIC), coupled to each other via dipole–dipole interactions. Interactions due to transition dipole moments lead to the formation of delocalized excitonic modes, represented as \(|k_i\rangle\) in Figure 1b. The presence of permanent dipole moments both in the ground and excited molecular states leads to the emergence of two-exciton states, with a large electric field per photon due to microm-size confinement and an external weak probe field of frequency \(\omega_p\) that is far-detuned from the lowest cavity resonance frequency by an amount comparable to \(U_i\); but nevertheless propagates inside of the microcavity due to the finite transmittivity of the upper lossy mirror.

We wish to exploit the organic medium as a nonlinear optical material with a large optical response at low light levels. In particular, we want to demonstrate that by exploiting the strong dipole–dipole interaction between individual chromophores due to their permanent dipoles, plus the strong collective coupling of a molecular aggregate with the cavity field, it is possible to perform light-by-light switching at the single-photon level. In this Letter, we develop a semiclassical model to predict that the presence of a single photon (on average) at the cavity frequency can modify absorptive and dispersive response of the organic medium to a weak external probe at a different frequency. The intermolecular electronic coupling between chromophores is responsible for establishing the required anharmonicities in the material spectrum, and the large electric field per photon of the confined cavity mode reduces the number of control photons required to achieve an observable switching effect. The model is semiclassical in the sense that we neglect, for simplicity, all quantum correlations between the cavity mode and the J-aggregates. However, we need to consider the quantized nature of the cavity and probe fields due to the low mean photon numbers (less than two) involved in the scheme.

To describe the evolution of the medium polarization \(\mathbf{P}(t)\), we employ a quantum Langevin formalism. The key features of our model are (i) the strong coupling of the molecular ensemble with the cavity field and (ii) the intermolecular resonant energy transfer via transition dipoles, known as Förster coupling \(J_p\), in addition to diagonal dipole–dipole interaction \(U_i\) via permanent dipoles. Additionally, we consider chromophore relaxation due to spontaneous emission outside of the confined cavity mode, coupling of the chromophores to a phonon bath, and inhomogeneous broadening due to static disorder in chromophore transition energies. The evolution of an observable \(O\) in the Heisenberg picture is given by \(dO/dt = -[O, \mathcal{H}_f + \mathcal{H}_S]\) (we use \(\hbar = 1\) throughout), where the Hamiltonian \(\mathcal{H}_f\) describes the coherent evolution of the system degrees of freedom and \(\mathcal{H}_S\) the interaction of the system with the environment. More specifically, \(\mathcal{H}_S\) describes the interaction of a single planar J-aggregate containing \(N\) chromophores with the electromagnetic field of a single cavity mode at frequency \(\omega_c\), as well as a probe field at frequency \(\omega_p\) and can be partitioned as

\[
\mathcal{H}_S = H_l + H_2 + H_3
\]

The first term describes a single effective molecular aggregate, as defined in the Supporting Information (SI), in the one-exciton eigenbasis as

\[
H_l = \sum_k \omega_k B_k^\dagger B_k + \sum_{k\neq p} U_{kp} B_k^\dagger B_p
\]

The bosonic operator \(B_k\) annihilates an exciton in the \(k\)th mode, with \(k = \{1, 2, \ldots, N\}\). We assume that the aggregate is a collection of two-level chromophores, with ground state \(|g\rangle\) and excited state \(|e\rangle\) having a site-dependent transition energy \(\epsilon_i = \omega_c + d_i\), where \(d_i\) is a small random shift from the gas-phase transition frequency \(\omega_c\) that models structural or so-called static

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**Figure 1.** (a) Illustration of an optical microcavity containing an ensemble of two-dimensional J-aggregates. The cavity mode \(a_{c}\) is driven by a weak input field \(a_{in}\) and decays through the semireflecting mirror at the rate \(\gamma_c\). A weak probe field at frequency \(\omega_p\) couples directly to the organic chromophores. Individual molecules decay into external modes with a rate \(\gamma_e\). Dipole–dipole interactions between individual chromophores in each aggregate modify the single-molecule response of the medium to the cavity and probe fields. (b) Energy spectrum of an individual aggregate showing the one-exciton and two-exciton bands (bandwidths not on scale). The cavity field drives all of the allowed coherences between states \(|k_i\rangle\) and \(|k_j\rangle\), and the weak probe (in the ground state \(|g\rangle\). The transition frequency between \(|k_i\rangle\) and \(|k_j\rangle\) is shifted by the interaction energy \(\sim U_i\) with respect to the noninteracting case. (c) Effective four-level system coupled by the cavity with Rabi frequencies \(\Omega_2 > \Omega_3 > \Omega_1\). The cavity frequency \(\omega_c\) is assumed to be near-resonance with the transition \(|k_1\rangle\rightarrow|k_2\rangle\). This model is used in eq 8 to describe the probe absorption at frequency \(\omega_p < \omega_c\).
The first term in eq 2 is the diagonal form of the site-basis Frenkel exciton Hamiltonian $H_0 = \sum e_i B_i^\dagger B_i + \sum_{ij} J_{ij} B_i^\dagger B_j$. We take the electrostatic interaction between molecules as purely classical. In the point dipole approximation, the exchange coupling energy is $J_{ij} = (1 - 3 \cos^2 \Theta_{ij}) \frac{e^2}{r_{ij}^3}$, where $\Theta_{ij}$ is the angle between the transition dipole moments $d_{iq}$ of molecules $i$ and $j$ (assumed parallel) and the intermolecular separation vector $r_{ij} = r_i - r_j$. The second term in eq 2 describes the interaction between two exciton eigenstates due to long-range forces between the permanent dipoles of the chromophores. Here, we assume a simplified form of the scattering potential $U_{ij} = \sum \epsilon_{ikq} \ket{kq} \bra{ik}$, where $\epsilon_{ikq}$ is an element of the unitary transformation $B_i = \sum_k \epsilon_{ikq} B_k$. The interaction energy between sites is $U_{ij} = (1 - 3 \cos^2 \Theta_{ij}) (\Delta d)^2 / r_{ij}^3$, where $\Delta d = d_i - d_j$ is the change in permanent dipole moment upon excitation of the chromophores. For homogeneous aggregates, large values of $U_{ij}$ can lead to the formation of bieexcitons with a binding energy proportional to $U_{ij}$.  

In this work, we simplify the two-exciton problem by assuming that the leading effect of the potential $U_{ij}$ is to red shift or blue shift the two-exciton band with respect to the noninteracting case, for attractive or repulsive interactions, respectively. For simplicity, we take two-exciton eigenstates as direct products of single-exciton states.

The second term $H_2 = \omega_p a_1^\dagger a_1 + \omega_a \mathcal{E}(t) E$ in eq 1 is the free Hamiltonian for the cavity and probe fields, and the third term describes light--matter interaction as

$$H_3 = i \sum_{kq} g_k(t)(\mathcal{E} B_k^\dagger - B_k^\dagger \mathcal{E}) + i \sum_{kq} D_{kq}(a_1^\dagger B_k^\dagger B_q - B_k^\dagger B_q^\dagger B_k a)$$

where $g_k(t) = (N_A \epsilon)^{1/2}(\mu_k \cdot \mathbf{e}_k)E(t)$ is proportional to the single-exciton transition dipole moment $\mu_k$, $\epsilon = \mu_k \cdot \mathbf{e}_k$ and $D_{kq} = (N_A \epsilon)^{1/2}(\mu_k \cdot \mathbf{e}_k)\mathcal{E}_k$ is proportional to the one-to-two-exciton transition dipole moment $\mu_{kq} = \ket{kq} \bra{kq}$. We assume that the cavity supports a single mode with transverse polarization that is near-resonance with the strongest one-to two-exciton transition. Higher-frequency modes are assumed to be far-detuned and thus ignored. The organic medium typically consists of an ensemble of aggregate domains. Within each domain, the intermolecular interactions are much stronger than those between domains. For simplicity, we idealize the medium by assuming that each domain contains a single one-dimensional aggregate, and all domains are identical. $N_A$ is the number of aggregates in the medium (details in the SI). $E(t)$ and $\mathbf{e}_k$ are the electric field envelope and polarization of the probe. $\mathcal{E}_k$ is the electric field per cavity photon and $\mathbf{e}_k$ its polarization. The probe and cavity polarizations are assumed to be collinear. Each molecular aggregate interacts simultaneously with the cavity and probe fields. In eq 3, we assume that the probe and cavity fields do not drive the same transitions. In reality, both fields interact with all allowed transitions, but we neglect off-resonant contributions to the light--matter coupling. The system--bath interaction is partitioned as $\mathcal{H}_{SB} = H_{SB} + H_{cav}$, where $H_{SB}$ describes the decay of the cavity mode through the mirror of a one-sided microcavity, which corresponds to a typical experimental setup.  

The term $H_{SB}$ describes the radiative decay of excitons into electromagnetic modes outside of the cavity in addition to dephasing of excitons via interactions with phonons. The specific form $\mathcal{H}_{SB}$ and the relaxation tensors for system observables used in this work are given in the SI. We are interested in the polarization $P(t)$ of the medium, induced by the weak coherent probe field $E$, with frequency $\omega_p$. The medium polarization at frequency $\omega_p$ is given by

$$P(t) = \sum_k \mu_k \langle \langle B_k(t) \rangle + \langle B_k^\dagger(t) \rangle \rangle$$

We therefore need to solve the quantum Langevin equation for the excitation coherence in the steady state $\langle B_k(t \rightarrow \infty) \rangle$. The nonlinearity in the system Hamiltonian $H_0$ in eq 1 couples the observable $B_k$ with an infinite hierarchy of equations of motion involving powers of the material operators $B_k$ and $B_{k'}$. Because we are interested in the interaction of the medium with at most one probe photon and one cavity photon on average, we invoke a dynamics-controlled truncation scheme (DCT) to truncate the hierarchy at third order, thus neglecting correlation functions involving products of four or more excitonic variables in the equations of motion. Given the small excitation density generated by the weak probe, we also assume that the ground-state population remains near unity at all times, ignoring contributions from density terms such as $\langle B_k B_{k'} \rangle$ in the equations of motion. Moreover, we assume a semiclassical approximation for the cavity field, which amounts to factorizing the correlation functions involving products of cavity and material variables. The dynamics of the medium polarization is thus governed by the equations

$$\frac{d}{dt} \langle B_k \rangle = \left(-i\omega_k - \frac{\Gamma_k}{2} \right) \langle B_k \rangle - g_k \langle \mathcal{E} \rangle$$

$$\frac{d}{dt} \langle B_k^\dagger B_q \rangle = \left[-i(\omega_p + \omega_q + 2U_{pq}) - \Gamma_{pq} \right] \langle B_p B_q \rangle$$

$$\langle B_k^\dagger B_q \rangle = -2 \sum_{p} D_{kq}(a_1^\dagger B_p) - \langle g_p \langle B_q \rangle + g_q \langle B_p \rangle \rangle \langle \mathcal{E} \rangle$$

Assuming that the phonon and the photon baths are Markovian, the Langevin noise terms in the equations of motion do not contribute to the evolution of the expectation values $\langle B_k \rangle$ and $\langle a_1 \rangle$. For simplicity, we have also neglected the effect of Langevin noise terms in the two-point and three-point correlation functions. For the validity of the semiclassical cavity regime, we assume that the cavity mode is weakly driven by a coherent field with $\langle a_1 \rangle > 0$ in the underdamped regime. We have ignored contributions of three-point correlation functions of the form $\langle B_k^\dagger B_q B_p \rangle$, representing one-to-two exciton coherences. These coherence can be shown to remain negligible unless the ground state is depleted by the probe field beyond the perturbative regime. The cavity field couples directly to the $\langle B_k^\dagger B_q \rangle$ in the Langevin equations (see the SI). Therefore, in the perturbative regime with respect to the probe field, the cavity amplitude $\langle a_1(t) \rangle$ evolves and is not affected. In other words, the cavity field does not undergo Rabi oscillations. Despite the number of simplifications made in the derivation of eqs 5 and 6, we note that they have the same structure as the NEE that Chernyak et al. derived by taking into account the nonboson commutation of exciton operators, density terms, and inelastic exciton--exciton scattering. Therefore, on the basis of several previous studies of nonlinear optical spectroscopy
using the NEE in the regime of perturbative light–matter interaction,\textsuperscript{20} we expect our model to provide a accurate qualitative description of the nonlinear response of molecular aggregates in microcavities where the coupling to the cavity mode is nonperturbative.

We are interested in the steady-state response of the system to the probe field for time scales long compared with the exciton and cavity lifetimes. Moreover, we assume that the cavity field decays at a rate slower than the exciton coherence decay (1/\(\tau_c\) \textasciitilde 10\(^2\) fs), which does not require very high Q cavities at room temperature.\textsuperscript{17} This separation of time scales allows us to solve eqs 5 and 6, taking the cavity and probe amplitudes as constants. Another important assumption in our model is the resolution of frequencies in the system. We require the detuning of the probe field \(\delta = \omega_p - \omega_e\) to be larger than the exciton line width. In the SI, we show that under such assumptions, we can use eqs 5 and 6 to write the steady-state probe susceptibility as

\[
\chi_p(\omega_p) = \sum_a \mu_a X_a / \Gamma_p
\]

where \(\mu_a = \langle \mu \rangle \) and the one-exciton coherences \(X_a = [X_1, X_2, ..., X_N]^T\) are obtained by solving the linear system

\[
MX = B
\]

where the matrix \(M = (O + 2\Delta \mu \mu^T)\) encodes the relevant light–matter interaction parameters in the one-photon and two-photon detuning matrices \(O\) and \(T\), respectively, and \(A_i \equiv \langle a \rangle\) is the mean cavity amplitude. The vector \(B = [\mu_1, \mu_2, ..., \mu_N]^T\) contains the one-exciton transition dipole moments. The definition of the matrices \(O\) and \(T\) can be found in the SI.

In the adopted semiclassical regime, the absence of the cavity field is defined as \(A_i = 0\), where the linear response is simply given by a sum of Lorentzians centered at the exciton frequencies \(\omega_0\) weighted by the corresponding transition dipole moments \(\mu_i\). The coupling to the cavity therefore modifies the absorptive and dispersive response of the medium to the weak probe, as described below. We note that the aggregate absorption spectra obtained from eq 7 satisfies the sum rule \(\int \Im[\chi(\omega_p)] d\omega_p = N\langle \mu \rangle / \epsilon_0\) for all values of \(A_i\).

In order to illustrate the developed model in eq 7, we calculate the probe susceptibility \(\chi_p(\omega_p)\) for open one-dimensional homogeneous J-aggregates of size N. In Figure 2, we show the computed absorption probe spectra for PIC J-aggregates with \(N = 100\) and 6. We use the transition energy \(\omega_e = 2.25\) eV for all sites, nearest-neighbor excitonic coupling \(J_2 = -68.2\) meV, and dipole–dipole coupling \(U_{12} = -198\) meV. These parameters were obtained by Markov et al.\textsuperscript{36} from the observation of a red-shifted induced absorption peak in the pump–probe spectra of PIC aggregates. The relaxation tensors \(\Gamma_1\) and \(\Gamma_p\) are dominated by phonon scattering (see SI), and for simplicity, we set \(\Gamma_1 = \Gamma_p \approx 26\) meV, which gives a lower bound for the exciton dephasing rate at room temperature.\textsuperscript{37}

The vacuum Rabi frequency in microcavities can reach values on the order of \(\Omega_c \approx 100\) meV.\textsuperscript{7} In Figure 2, we set the vacuum Rabi frequency \(\Omega_c \equiv (N\langle \mu \rangle) \sqrt{\langle \mu \rangle^2} / \epsilon_0\epsilon_0\) so that \(D_{k,p} \approx N^{3/2} \Omega_c^3\) for the strongest excitonic transitions exceeds the dissipation rates, as is required in the strong coupling regime.

The probe absorption spectra in Figure 2a display a four-peak structure where the peak splitting scales linearly with the mean cavity amplitude \(A_c\). The free-space spectrum corresponds to the J-band without static disorder. The peak splitting trend with increasing \(A_c\) can be qualitatively explained using a semiclassical model in which a classical cavity field of frequency \(\omega_e\) couples strongly with two states in the one-exciton band (labeled \(l_{k_1}\) and \(l_{k_2}\)) and two states in the two-exciton band (\(l_{k_3}\) and \(l_{k_4}\)). The coupling scheme is illustrated in Figure 1c. The transition dipole moments from the ground state \(|g\rangle\) to the states \(l_{k_1}\), \(l_{k_2}\), and \(l_{k_3}\) have the largest values in the one-exciton band and satisfy \(\mu_1 > \mu_2 > \mu_3\). The cavity frequency is chosen to be on resonance with the \(l_{k_1} \rightarrow l_{k_3} k_4\) transition. The effective Hamiltonian \(H_{eff}\) that describes the couplings between the two bands in the rotating frame of the cavity field is given by

\[
(H_{eff}|_{L2}) = \begin{pmatrix}
\mu_1 & \mu_2 & 0 & 0 \\
\mu_2 & \mu_3 & 0 & 0 \\
0 & 0 & \mu_3 & \mu_4 \\
0 & 0 & \mu_4 & \mu_5
\end{pmatrix}
\]
where the frequency parameters are defined in Figure 1c. Energy is measured with respect to the lowest exciton state \( |\epsilon_1\rangle \), that is, \( \omega_1 \equiv 0 \). For a large homogeneous aggregate, the excitonic bands become quasi-continuous, and the splittings \( \Delta_2 \equiv \omega_2 - \omega_1 \) and \( \Delta_{23} \equiv \omega_3 - \omega_1 \) become negligibly small in comparison with typical line widths. Therefore, we can assume that the cavity strongly couples almost on resonance for the four levels shown in Figure 1c for large aggregates. In this regime, a weak probe field will drive transitions between the ground state \( |g\rangle \) and the eigenstates of the effective Hamiltonian \( \mathcal{H}_{\text{eff}} \) in eq 8, which are the new normal modes of the cavity–molecule system. In the inset of Figure 2a, we show the eigenvalues of \( \mathcal{H}_{\text{eff}} \) for \( \omega_1 = \omega_{12} = \omega_5 \), \( \Delta_2 = 0 \), \( \Omega_3 = 3 \Omega_1 \), and \( \Omega_2 = \Omega_3/3 \) as a function of \( \Omega_5 \equiv \Omega_2 \). For \( \Omega_5 \ll 1 \), only three peaks can be resolved, but as \( \Omega_5 \) increases, the middle peak splits into a doublet, giving rise to the four-peak structure observed in the probe spectra calculated using eq 7, which includes \( N \) one-exciton states and \( \sim N^2 \) two-exciton states.

The number of states in the one- and two-exciton bands that couple strongly with the cavity field depends on the size of the molecular aggregate. In order to illustrate this fact, we show in Figure 2b the probe absorption spectrum for an open 1D homogeneous aggregate of size \( N = 6 \) without static disorder, with the same values of \( \omega_0 = \omega_{12} \), \( U_{12} \), and decay parameters as in Figure 2a. The cavity field is again resonant with the \( |k_1 \rangle \rightarrow |k_2 \rangle \) transition, but now, the states \( |k_1 \rangle \) and \( |k_2 \rangle \) are no longer quasi-degenerate with \( |k_3 \rangle \) because of the smaller array size. The cavity frequency is thus detuned from their corresponding transitions with the states \( |k_1 \rangle \) and \( |k_2 \rangle \) in the two-exciton band. Because the Rabi frequency \( \Omega_3 \) is proportional to the transition dipole \( \mu_{k_1,k_2} \) by construction, whenever \( \Omega_3/\Delta_{23} \ll 1 \), we can set \( \Omega_3 = 0 \) in eq 8 to effectively remove the state \( |k_3 \rangle \) from the excited-state dynamics. Interestingly, the eigenstates of the resulting three-level system with \( \Delta_2 > 0 \), plotted as a function of \( \Omega_5 \) in the inset of Figure 2b, show a trend in very good agreement with the microscopic model derived in eq 7.

We now include the effect of inhomogeneous disorder in the evaluation of the probe absorption spectra. In order to model static energetic disorder, we assume that the site energies are given by \( \epsilon_i = \omega_0 + \delta_i \) where \( \delta_i \) is a random energy shift taken independently for each site from a Gaussian distribution with standard deviation \( \sigma/\mu \approx 0.1 \), consistent with the motional narrowing limit.\(^{30,31} \) The susceptibility \( \chi' \) obtained from eq 7 needs to be averaged over the ensemble of disorder realizations. In Figure 3, we show the probe absorption spectrum \( \mathcal{A}(\omega_p) \) of the same J-aggregates used in Figure 2a with \( N = 100 \) and 6 molecules, but now, static disorder is introduced in the Hamiltonian. The structure of the spectrum resembles the results in Figure 2a, but the details of the splittings near the origins are different because now the detunings \( \Delta_2 \) and \( \Delta_{23} \) in eq 8 are averaged over an ensemble of disorder realizations. The behavior of the outer peaks persists in the presence of disorder as a result of the strong interaction between the cavity mode and the transition \( |k_1 \rangle \rightarrow |k_2 \rangle \), close to the deterministic resonance frequency \( \omega_{12} = \omega_3 - 2U_{12} \).

As a final example, we consider the response of a dimer of coupled polar chromophores.\(^{38} \) Clearly for \( N = 2 \), the bosonic approximation used in the derivation of eq 7 is no longer valid to describe the two-exciton manifold, which now consists of a single state \( |\epsilon_2 \rangle \). However, because the structure of eqs 5 and 6 is universal, the steady-state solution for the one-exciton manifold has states \( |\epsilon_1 \rangle \), \( |\epsilon_1 \pm \epsilon_2 \rangle \), \( |\epsilon_2 \rangle \), \( |\epsilon_1 \pm a|\epsilon_2 \rangle \), and \( |\epsilon_1 \rangle \rightarrow |\epsilon_1 \epsilon_2 \rangle \), \( |\epsilon_1 \rangle \rightarrow |\epsilon_1 \epsilon_2 \rangle \), \( |\epsilon_2 \rangle \rightarrow |\epsilon_2 \epsilon_2 \rangle \), \( |\epsilon_1 \rangle \rightarrow |\epsilon_1 \epsilon_2 \rangle \), \( |\epsilon_2 \rangle \rightarrow |\epsilon_1 \epsilon_2 \rangle \), and \( |\epsilon_2 \rangle \rightarrow |\epsilon_2 \epsilon_2 \rangle \). Given that the two-exciton energy \( \omega_{12} = \epsilon_1 + \epsilon_2 - U_{12} \) is red shifted with respect to the one-exciton transition frequencies \( \omega_{\epsilon_2} \), the response of a single dimer to a probe field at frequency \( \omega_p \) when the cavity is resonant with the \( |\epsilon_2 \rangle \rightarrow |\epsilon_1 \epsilon_2 \rangle \) transition, is given by

![Figure 3. Probe absorption \( \mathcal{A}(\omega_p) \) for a 1D J-aggregate as a function of the probe detuning from an arbitrarily chosen one-exciton state near the bottom of the one-exciton band. Dipolar couplings \( \mathcal{J}_i \) and \( \mathcal{U}_i \) are the same as those in Figure 2a. The cavity frequency is chosen such that the two-photon detuning vanishes when \( \omega_p - \omega_1 = 0 \). (a) The aggregate size is \( N = 100 \). Curves are labeled according to the mean cavity amplitudes: (a) no cavity, (b) \( A_1 = \langle |a_1| \rangle = 0.1 \), (c) \( A_1 = 0.2 \), and (d) \( A_1 = 0.3 \). (b) The aggregate size is \( N = 6 \) for (a) no cavity, (b) \( A_1 = 0.4 \), (c) \( A_1 = 0.8 \), (d) \( A_1 = 1.2 \), (e) \( A_1 = 1.6 \), and (f) \( A_1 = 2.0 \). In both panels, the vacuum Rabi frequency is \( \Omega_k = \Gamma \), where \( \Gamma = 26 \) meV is the exciton decay rate. Static disorder is modeled by taking each monomer energy randomly from a Gaussian distribution with mean \( E_0 = 2.25 \) eV and standard deviation \( \sigma = 0.125 \) eV.

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which corresponds to eq 7 in the limit where $M$ has a single nonzero element. Here, $\Delta_p = \omega_p - \omega_i$, $\Delta_c = \omega_c - (\epsilon_1 + \epsilon_2 + U_1 - \omega_i)$, $\mu_i = \mu_c$, and $D_{12} \gg \mu_c$. $\Gamma_1$ and $\Gamma_{12}$ are the one- and two-exciton decoherence rates. Equation 9 shows that for coupled dimers, the cavity acts as a control for the propagation of the weak probe, in analogy with the phenomenology stemming from atomic physics to describe EIT in a cascaded three-level system.13 Figure 4 shows the absorptive and probe susceptibility has the standard features of EIT, reduction of polar chromophores in panels a and b, respectively. The static disorder because the two-exciton coherence is short-lived, can assemble into low-dimensional aggregate structures. We developed a theoretical model to analyze and interpret nonlinear optical signals in the regime of strong cavity–matter coupling. The constructed model is constrained within the current experimental capabilities, and we illustrate our findings using experimentally reported parameters for PIC polar dyes. The conclusions of our work are however general and go beyond our specific choice of molecular species and microcavity configuration. Organic chromophores with large permanent dipoles $d \approx 1–10 \text{D}$, both in ground and excited states, continue to be under active experimental investigation for the design of second-order nonlinear optical materials.40–42 Upon aggregation, these polar dyes can lead to strong exciton–exciton interactions that exceed the broadening of the exciton

\[ \chi(\omega_i) = \left[ \frac{\epsilon_1^2}{\epsilon_0} \right] \left( \frac{\Gamma_{12} - i(\Delta_p + \Delta_c)}{(i\Delta_p - \Gamma_1)(i\Delta_c - \Gamma_{12}) + 2D_{12}^2|A_c|^2} \right) \]  

(9)

\[ A_{\text{min}} = \frac{\Gamma_1 + \gamma_p + \gamma_c}{\Gamma_{12} + \gamma_p + \gamma_c + 2D_{12}^2|A_c|^2} \]  

(10)

Figure 4. Im[$\chi(\omega_i)$] (a) and Re[$\chi(\omega_i)$] (b) for an inhomogeneously broadened dimer as a function of the probe detuning from the maximum of the free-space exciton absorption band. Dipolar couplings $J_{12}$ and $U_{12}$ are the same as those in Figure 2. The cavity frequency is chosen such that the two-photon detuning vanishes when $\omega_p - \omega_i = 0$. Curves are labeled according to the mean photon number $A_c = \langle n \rangle$: (a) no cavity, (b) $A_c = 0.5$, and (c) $A_c = 1.0$. The vacuum Rabi frequency is $\Omega = 5\Gamma$, where $\Gamma = 26 \text{meV}$ is the exciton decay rate. (c) Minimum absorption $A_{\text{min}}$ (normalized to the free-space value) near the deterministic resonance $\omega_p - \omega_i = 0$ (between the Autler–Townes doublet) as a function of the mean cavity amplitude $A_c$. Curves are labeled by the width of the Gaussian distribution of static energy shifts $\sigma$ (in units of the exchange dipole coupling $f$). All other parameters are the same as in those panels a and b.
plasmonic nanostructures, which further opens the possibility of quantum control of excited-state dynamics. In this regime, it should be possible to control the outcome of chemical reactions at the level of thermodynamics by effectively lowering reaction barriers, in analogy with traditional catalytic processes, thus directly affecting reaction kinetics. This novel strong-field single-photon quantum control paradigm for molecular processes should be contrasted with traditional strong-field laser control schemes that exploit delicate laser-induced quantum interferences among internal vibronic states but do not modify the energetics of the reaction. Quantum optical control of chemical dynamics is a future research direction with promising applications in nanoscience and technology, where traditional bulk methods for controlling chemical reactivity have limited efficiency.

**ASSOCIATED CONTENT**

# Supporting Information

The derivation of the quantum Langevin equations leading to eqs 5 and 6 and the definition of the dynamical matrix M in eq 7 can be found. This material is available free of charge via the Internet at http://pubs.acs.org.

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Notes

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